Single-frequency oscillation of thin-disk lasers due to phase-matched pumping

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Abstract: We present a novel pump concept that should lead to single-frequency operation of thin-disk lasers without the need for etalons or other spectral filters. The single-frequency operation is due to matching the standing wave pattern of partially coherent pump light to the standing wave pattern of the laser light inside the disk. The output power and the optical efficiency of our novel pump concept are compared with conventional pumping. The feasibility of our pump concept was shown in previous experiments.

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References and links

1. Introduction

Single-frequency oscillation of Yb:YAG thin-disk lasers without etalons or other spectral filters is usually limited to low output powers because above a certain pump power additional longitudinal modes reach the lasing threshold and oscillate simultaneously. It seems that the highest reported single-frequency output power of a thin disk laser is 30 W [1]. Single-frequency oscillation and wavelength tuning was achieved by placing a Lyot filter in the laser cavity. In this paper, we present a pump concept for thin-disk lasers that leads to single-frequency emission without etalons or spectral filters. Additionally, the emission wavelength can be tuned in a range of some nanometers by slightly changing the pump arrangement. Since the oscillation wavelength is tunable, the output power of several thin-disk lasers can be combined by a dispersive element to increase the brightness while preserving the spatial beam quality. First experimental results using an intra-cavity pumped thin-disk laser using this pump concept demonstrated single lon-
itudinal mode operation of a thin-disk laser and wavelength tunability of the lasing mode [2]. Thin-disk lasers use an active medium that has the shape of a disk with a typical thickness $d$ of 100 $\mu$m to 300 $\mu$m and a diameter of several millimeters [3]. The front side of the disk is AR-coated for the pump light and the laser light and the back side of the disk is HR-coated. Thin-disk lasers employ multipass pumping schemes in order to absorb most of the pump light. The most common multipass pumping scheme consists of a parabolic mirror and roof prisms. This setup re-images the pump spot several times onto the disk [4]. Inside the disk, incident pump light and reflected pump light spatially overlap. If the pump light were fully coherent, a standing wave pattern with 100% contrast would be produced throughout the crystal. In the case of partially coherent pump light, the standing wave pattern has 100% contrast at the HR-coated back side of the disk and the contrast decreases with increasing distance from the back side. The decrease in contrast depends on the spatial beam quality and the coherence length of the pump light.

Recently, pump diodes became available for pumping the zero phonon line of Yb:YAG at 969 nm [5]. These pump diodes have an emission bandwidth of 0.6 nm. The small emission bandwidth is necessary because the width of the zero phonon line in Yb:YAG is only 0.5 nm [6]. The emission bandwidth of 0.6 nm corresponds roughly to a coherence length of $\ell_c = \lambda_p^2 / \delta \lambda \approx 1.6$ mm, where $\lambda_p$ is the diode wavelength and $\delta \lambda$ is the emission bandwidth. This coherence length is roughly twelve times larger than a 130 $\mu$m thick disk. As a consequence, the incident pump light and the reflected pump light produce a standing wave pattern throughout the disk with high contrast.

The distance the pump light has to travel between successive double-passes is typically many centimeters and thus much longer than the coherence length. Therefore the intensity that results from the superposition of all double-passes exhibits temporal fluctuations due to the irregular phase relationship between pump light belonging to different double-passes. However, these fluctuations are on the gigahertz scale and therefore irrelevant for the much slower gain dynamics. Note that although the magnitude of the intensity is fluctuating, the period and the position of the standing wave pattern is stationary.

We now show that the standing wave pattern of the pump light can be matched to the standing wave pattern in the laser medium.
wave pattern of the laser light. For the moment, we assume that the pump light is a monochromatic plane wave and we neglect absorption. We also assume that the HR coating at the back side of the disk induces the same phase shift on reflection for the laser light and the pump light and that this phase shift is equal for s-polarized light and of p-polarized light. Our assumptions on the phase shifts are usually valid for broadband HR coatings. The pump light hits the disk at an angle \( \theta_0 \) with respect to the surface normal and propagates at an angle \( \theta \) inside the disk. The angle \( \theta \) is related to the angle of incidence \( \theta_0 \) via Snell’s law \( \theta = \sin^{-1} \left( \sin \theta_0 / n_p \right) \), where \( n_p \) is the refractive index of the disk for the pump light. The intensities of the s-polarized standing wave pattern \( I_p^{(s)} \) and the p-polarized standing wave pattern \( I_p^{(p)} \) of the pump light are given by [7]

\[
I_p^{(s)}(z) = 4I_{p,0}^{(s)} \sin^2 \left( k_p z \cos \theta \right), \quad \text{(1)}
\]

\[
I_p^{(p)}(z) = 4I_{p,0}^{(p)} \left[ \sin^2 \left( k_p z \cos \theta \right) \cos^2 \theta + \cos^2 \left( k_p z \cos \theta \right) \sin^2 \theta \right], \quad \text{(2)}
\]

where \( I_{p,0}^{(s)} \) and \( I_{p,0}^{(p)} \) are the intensities of the incident s-polarized and p-polarized pump light of the first pass, \( k_p \) is the angular wavenumber of the pump wavelength in the disk given by \( k_p = 2 \pi n_p / \lambda_p \), and \( z \) is the position along the axis of the disk. The grating period of the standing wave pattern is \( \Lambda_p = \lambda_p / (2 n_p \cos \theta) \). The standing wave pattern of the s-polarized pump light is shown in Fig. 1.

We restrict the discussion to I-resonators, i.e. a resonator with a disk and a mirror, creating a straight, unfolded beam such that disk, beam, and mirror together resemble the letter “I”. In an I-resonator the standing wave pattern of the laser light in the disk is given by

\[
I_L(z) = 4I_{L,0} \sin^2 \left( k_L z \right), \quad \text{(3)}
\]

where \( I_{L,0} \) is the circulating intensity inside the resonator, \( k_L \) is the angular wavenumber of the laser light given by \( k_L = 2 \pi n_L / \lambda_L \), \( n_L \) is the refractive index for the laser light, and \( \lambda_L \) is the laser wavelength in vacuum. The standing wave pattern of the laser light has the grating period \( \Lambda_L = \lambda_L / (2 n_L) \). The grating period of the laser light \( \Lambda_L \) and the grating period of the pump light \( \Lambda_p \) are phase-matched when the trigonometric argument \( k_p z \cos \theta \) in Eq. (1) and Eq. (2) is equal to the trigonometric argument \( k_L z \) in Eq. (3). For p-polarized light, this condition only leads to optimum phase matching, not to perfect phase matching because the second, weak \( \sin^2 \theta \) term in Eq. (2) is out of phase with respect to the strong first \( \cos^2 \theta \) term. Solving for \( \theta \) gives the phase matching angle inside the disk \( \theta_{PM} \):

\[
\theta_{PM} = \cos^{-1} \left( \frac{\Lambda_p n_L}{\Lambda_L n_p} \right). \quad \text{(4)}
\]

According to Snell’s law the angle of incidence of the pump light on the disk \( \theta_{0,PM} \) to achieve phase matching is given by

\[
\theta_{0,PM} = \sin^{-1} \left( \frac{1}{\Lambda_L} \sqrt{\Lambda_L^2 n_p^2 - \Lambda_p^2 n_L^2} \right). \quad \text{(5)}
\]

At a pump wavelength of 969 nm and a laser wavelength of 1030 nm, the angle of incidence of the pump light that leads to phase matching is \( \theta_{0,PM} \approx 38^\circ \). Using this angle of incidence, the medium is not pumped at the nodes of the standing wave pattern of the laser light. These nodes stabilize single-frequency emission if a quasi-three-level medium is used for the disk [8].
Unpumped regions in a quasi-three-level medium represent a loss. A mode whose standing wave is not phase matched to the standing wave of the pump light would have a non-vanishing amplitude of its electrical field at these unpumped regions.

2. Requirements on the pump diodes

Our pump concept requires a standing wave pattern with high contrast inside the disk. The contrast is reduced when the pump light is not polarized, the emission bandwidth of the pump diodes leads to a coherence length that is shorter than the disk’s thickness, and when the spatial beam quality of the pump diodes is poor. We discuss these three influences separately. The Michelson contrast $K = (I_{\text{max}} - I_{\text{min}})/(I_{\text{max}} + I_{\text{min}})$ is used as a figure of merit for the contrast.

2.1. Polarization

Conventional thin-disk lasers are pumped by unpolarized light. Hence the diode pump intensity $I_p$ is made up of equal contributions of s-polarized and p-polarized light $I_p(z) = I_p^s(z)/2 + I_p^p(z)/2$ with $I_p^s = I_p^p$ in the temporal average. According to Eq. (1) and Eq. (2), the intensity at a node of the combined standing wave pattern of the two polarizations is

$$I_{\text{min}}(\theta) = 2I_p^0 \sin^2 \theta$$

and the largest intensity is $I_{\text{max}}(\theta) = 4I_p^0$. Thus, the Michelson contrast for unpolarized pump light at an angle of incidence $\theta_0$ onto the disk is

$$K_{\text{unpol}} = \frac{2}{\sin^2 \theta + 1} - 1 = \frac{2}{n_p^2 \sin^2 \theta_0 + 1} - 1. \quad (6)$$

At an angle of incidence of $\theta_{\text{PM}} = 38^\circ$ the contrast of the standing wave pattern reduces to $K_{\text{unpol}}(\theta_{\text{PM}}) = 0.8$ as shown in Fig. 2(a).

2.2. Emission bandwidth

The pump diodes must have a coherence length that is longer than the thickness of the disk which requires a sufficiently small spectral emission bandwidth. We call the emission bandwidth at which the contrast of the standing wave pattern has dropped to zero at the AR-coated side of the disk the critical emission bandwidth $\delta \lambda_{\text{crit}}$. The critical emission bandwidth is calculated for pump light with a top-hat spectral power density. The time averaged standing wave pattern for an emission bandwidth $\delta \lambda_p$ of the pump diodes is then approximately given by

$$I_{\delta \lambda_p}(z) = \frac{I_p}{\delta \lambda_p} \sin^2 \left[ \frac{2\pi n_p}{\lambda_p} z \cos \theta \right] d\lambda'$$

$$\approx \frac{1}{4} I_p \left\{ -2 \cos \left[ 2\pi n_p \left( \frac{1}{\lambda_p + \delta \lambda_p/4} - \frac{1}{\lambda_p - \delta \lambda_p/4} \right) z \cos \theta \right] \right. \left. \times \cos \left[ 2\pi n_p \left( \frac{1}{\lambda_p + \delta \lambda_p/4} + \frac{1}{\lambda_p - \delta \lambda_p/4} \right) z \cos \theta \right] + 2 \right\} \text{ for } \delta \lambda_p \leq \delta \lambda_{\text{crit}}, \lambda_p > \frac{\delta \lambda_p}{2}$$

$$\quad (8)$$

where $I_p$ is the pump intensity and $\lambda_p$ is the center wavelength of the pump light. The approximation that was made for deriving Eq. (8) is valid for bandwidths smaller than the critical bandwidth and for a pump wavelength larger than half the emission bandwidth. Eq. (8) is a beat between two spatial frequencies. The beat frequency is approximately given by
Fig. 2. Standing wave pattern of the pump light. In (a) random polarization of the pump light reduces the contrast of the standing wave pattern everywhere in the crystal. (b) An emission bandwidth of the pump diodes that is equal to the critical bandwidth $\delta \lambda_{\text{crit}}$ leads to a gradual decrease of the contrast starting from the HR coating at $z/d = 0$ towards the AR coating $z/d = 1$. (c) An angular distribution that is equal to the critical angular distribution $\delta \theta_{\text{crit}}$ leads to a similar gradual decrease of the contrast as in (b). The wavelength is exaggerated for the purpose of illustration.

Using this approximation, the envelope functions of Eq. (8) can be written as

$$I_{\delta \lambda, +\text{env}}(z) = \frac{1}{2} I_p \left[ \cos \left( \frac{\pi \delta \lambda_p}{\lambda_p^2 n_p \cos \theta} z \right) + 1 \right]$$

(10)

$$I_{\delta \lambda, -\text{env}}(z) = -\frac{1}{2} I_p \left[ \cos \left( \frac{\pi \delta \lambda_p}{\lambda_p^2 n_p \cos \theta} z \right) - 1 \right],$$

(11)

where $I_{\delta \lambda, +\text{env}}$ denotes the upper envelope and $I_{\delta \lambda, -\text{env}}$ denotes the lower envelope of the standing wave pattern given by Eq. (8). We can calculate the critical bandwidth $\delta \lambda_{\text{crit}}$ by setting Eq. (10) equal to Eq. (11) at $z = d$. This means the contrast is zero at the AR-coated side of the disk. The critical emission bandwidth is then given by

$$\delta \lambda_{\text{crit}} \approx \frac{\lambda_p^2}{2 d n_p \cos \theta} = \frac{\lambda_p^2}{2 d \left( n_p^2 - \sin^2 \theta_0 \right)^{1/2}} \text{ for } \lambda_p > \frac{\delta \lambda_{\text{crit}}}{2}. \quad (12)$$

The standing wave pattern inside the disk for an emission bandwidth equal to the critical bandwidth is shown in Fig. 2(b). The contrast of the standing wave pattern inside a disk made of Yb:YAG which is 130 $\mu$m thick drops to zero inside the disk, if the emission bandwidth of the pump diodes is equal to or larger than $\delta \lambda_{\text{crit}} = 3$ nm for a pump wavelength of 969 nm. If the emission bandwidth is smaller than $\delta \lambda_{\text{crit}}$, the contrast at the AR-coated side of the disk is

$$K_{\delta \lambda_p}(\delta \lambda_p, \delta \lambda_{\text{crit}}) \approx \cos \left( \pi d n_p \delta \lambda_p \cos \theta / \lambda_p^2 \right).$$

(13)

Currently, pump diodes are available having an emission spectrum of even less than 0.6 nm, which is roughly five times smaller than the critical bandwidth of a 130 $\mu$m thick disk.
2.3. Spatial beam quality

A full analysis for the required spatial beam quality of the pump diodes is beyond the scope of this paper but a simplified argument can be given by considering plane waves hitting the disk under slightly different angles of incidence. Each of these plane waves leads to a standing wave pattern with a slightly different grating period. In the time average these standing wave patterns can be summed up. The contrast of the resulting standing wave pattern gradually decreases from its maximum value at the HR-coated back side towards the AR-coated front side of the disk. We define the critical width of the angular distribution corresponding to the width of the angular distribution at which the contrast of the standing wave pattern has dropped to zero at the front side of the disk. We assume a monochromatic beam with a homogeneous distribution of power over the angles. The standing wave pattern for pump light having an angular distribution with a width of $\delta \theta$ is given by

$$I_{\delta \theta}(z) = \int_{\theta-\delta \theta/2}^{\theta+\delta \theta/2} I_p \left( \frac{2\pi n_p}{\lambda P} z \cos \theta' \right) \sin^2 \left( \frac{2\pi}{\lambda} n_p z \cos \theta' \right) d\theta'$$

$$\approx \frac{1}{4} I_p \left( -2 \cos \left( \frac{2\pi n_p}{\lambda P} \left[ \frac{1}{\cos (\theta + \delta \theta/4)} - \frac{1}{\cos (\theta - \delta \theta/4)} \right] z \cos^2 \theta \right) \right) \cos \left( \frac{2\pi n_p}{\lambda P} \left[ \frac{1}{\cos (\theta + \delta \theta/4)} + \frac{1}{\cos (\theta - \delta \theta/4)} \right] z \cos^2 \theta \right) + 2$$

for $\delta \theta < \delta \theta_{\text{crit}}$, $\theta > \delta \theta_{\text{crit}}/2$.

Again, the integral in Eq. (14) is approximately a beat between two close frequencies if the angle $\theta$ is larger than half the width of the angular distribution. Then the beat frequency is given by

$$f_{\delta \theta, \text{beat}} \approx \frac{n_p}{\lambda P} \delta \theta \sin \theta$$

The envelope functions of Eq. (15) are then given by

$$I_{\delta \theta, \text{env}}(z) = \frac{1}{2} I_p \left[ \cos \left( \frac{\pi}{\lambda P} n_p z \delta \theta \sin \theta \right) + 1 \right]$$

$$I_{\delta \theta, -\text{env}}(z) = -\frac{1}{2} I_p \left[ \cos \left( \frac{\pi}{\lambda P} n_p z \delta \theta \sin \theta \right) - 1 \right]$$

Then we calculate the critical angular distribution $\delta \lambda_{\text{crit}}$ by setting Eq. (17) equal to Eq. (18) at $z = d$. The critical angular distribution is then given by

$$\delta \theta_{\text{crit}} \approx \frac{\lambda_p}{2dn_p \sin \theta} = \frac{\lambda_p}{2d \sin \theta_0} \text{ for } \theta > \frac{\delta \theta_{\text{crit}}}{2}.$$
The contrast at the AR-coated side of the disk for an angle of incidence $\theta_0 > 0$ and angular distributions having a width smaller than $\delta \theta_{\text{crit}}$ is

$$K_{\delta \theta} \approx \cos\left(\frac{\pi \delta \theta_p d \sin \theta}{\lambda_p}\right) = \cos\left(\frac{\pi \delta \theta \sin \theta_0}{\lambda_p}\right) \text{ for } \theta > \delta \theta \text{ and } \delta \theta < \delta \theta_{\text{crit}}. \quad (20)$$

In a thin-disk laser the pump beam is usually focused onto the disk, creating a waist radius $w_p$ at the AR-coated side of the disk that is typically 30 to 50 times the thickness of the disk. Hence, a low beam parameter product is required. The minimum beam propagation factor $M^2$ [9] can be obtained that is necessary to achieve the critical angular distribution for a given radius of the pump spot $w_p$:

$$M^2_{\text{min}} \approx \frac{\pi}{\lambda_p} w_p^2 \frac{1}{2} \delta \theta_{\text{crit}} = \frac{\pi}{4 \sin \theta_0} \frac{w_p}{d}. \quad (21)$$

At a pump wavelength of 969 nm, a typical pump spot radius of 3 mm, an angle of incidence of $\theta_0 = 38^\circ$, and disk thickness of $d = 130 \mu$m the $M^2$ of the pump light should not exceed $M^2_{\text{min}} \approx 29$. At the moment fiber coupled pump diodes are available with a fiber core of 105 $\mu$m in diameter and a numerical aperture $NA = 0.22$. These pump diodes have an output power around 135 W and a $M^2 = \pi r_{\text{core}} NA / \lambda_p$ of $\approx 37$, where $r_{\text{core}}$ is the radius of the fiber core. The good spatial beam quality allows more pump passes through the disk hence the disk can be thinner and the required spatial beam quality is even lower. These pump diodes could be used in proof-of-concept experiments since they have an emission bandwidth of 0.7 nm in addition to the good spatial beam quality.

### 3. Analytical examination of output power and optical efficiency

In this section we analyze the stationary rate equations to compare the output power and the efficiency of thin-disk lasers which are pumped homogeneously and those pumped with a standing wave pattern. The laser operates in transverse multimode operation, so that the laser beam has the same diameter as the pump spot on the disk. Additionally, the resonance frequencies of the transverse modes differ only slightly from each other, i.e. we assume that different transverse modes have the same grating period as the fundamental mode. The pump spot is assumed to have a flat top intensity profile. Three different lasers are compared: The first laser is homogeneously pumped and the laser mode can extract the entire gain. This model corresponds to a zero-dimensional, monochromatic rate equation model. The second laser is pumped homogeneously as well but the gain is only extracted by a single longitudinal mode. This case corresponds to a conventionally pumped thin-disk laser that is forced to operate at a single longitudinal mode by using spectral filters. The third laser is pumped by a fully modulated standing wave pattern and the standing wave pattern of the pump is phase-matched to the standing wave pattern of the laser light. Quantities belonging to the first laser are denoted by $n_{\text{osHB}}$, to the second laser by $SHB$ and to the third laser by $PM$ meaning no spatial hole burning, spatial hole burning, and phase-matched, respectively. All three lasers are pumped with multiple pump passes. The lower case letter $i$ stands for the intensity normalized to the saturation intensity. The saturation intensity at the vacuum wavelength $\lambda$ is given by $I_{\text{sat}}(\lambda) = h c_0 / [\lambda (\sigma_{\text{abs}}(\lambda) + \sigma_{\text{em}}(\lambda)) \tau_{\text{us}}]$, where $\tau_{\text{us}}$ is the upper state lifetime, $h$ is Planck’s constant, $\sigma_{\text{abs}}(\lambda)$ and $\sigma_{\text{em}}(\lambda)$ are the effective cross sections for absorption and emission at the wavelength $\lambda$, and $c_0$ is the speed of light in vacuum.

The mean saturable absorption $\bar{\alpha} d$ at the pump wavelength of the three lasers is given by [10]...
\(Q\) where sections which we define as follows

For incoherent pump light the spatially averaged effective pump intensity is given by

The population of the ground state by introducing an effective cross section for absorption and

The absorption of the pump light is reduced. Above the lasing threshold, the ground state is

saturation intensity of the pump light. Pumping leads to a depletion of the ground-state and

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population of the ground state by introducing an effective pump intensity. The effective pump intensity is the product of the diode pump intensity \(i_{p,d}\) and the pump enhancement factor \(Q\).

For incoherent pump light the spatially averaged effective pump intensity is given by

\[
(\bar{a}d)_{noSHB} = \alpha_{p,0} \frac{1}{\alpha_{p,0} + 1} \frac{1}{1 + 2i_{L,0} + i_{p,d}Q_{noSHB}} d \tag{22}
\]

\[
(\bar{a}d)_{SHB} = \alpha_{p,0} \frac{1}{\alpha_{p,0} + 1} \frac{1}{1 + 2i_{L,0} + i_{p,d}Q_{SHB} + 4i_{L,0} + 4i_{p,d}Q_{SHB} + 4i_{L,0}} d \tag{23}
\]

\[
(\bar{a}d)_{PM} = \alpha_{p,0} \frac{1}{\alpha_{p,0} + 1} \frac{1}{1 + 2i_{L,0} + 2i_{p,d}Q_{PM} + 4i_{L,0} + 2i_{p,d}Q_{PM}} d \tag{24}
\]

where \(Q_{noSHB}, Q_{SHB}\) and \(Q_{PM}\) are pump enhancement factors (see Eq. (28), Eq. (29) and Eq. (30)). \(\alpha_{p,0}\) is the small-signal absorption coefficient for the pump light given by \(\alpha_{p,0} = \sigma_{abs,p}n_{dot}\) at the pump wavelength \(\lambda_p\), where \(\sigma_{abs,p}\) is the effective cross section for absorption and \(n_{dot}\) is the doping concentration of the disk. \(i_{L,0}\) is normalized to the circulating laser intensity inside the resonator and \(i_{p,d}\) is the normalized diode pump intensity, which is the intensity of the light emitted from the pump diodes when it first hits the disk.

\[
B := \frac{\sigma_{abs,p}}{\sigma_{abs,L}} \left[ \frac{\sigma_{em,L}}{\sigma_{abs,p} + \sigma_{em,p}} \right] \tag{25}
\]

where \(\sigma_{abs,p}\) and \(\sigma_{em,p}\) are the effective cross sections for absorption and emission at 969 nm and \(\sigma_{abs,L}\) and \(\sigma_{em,L}\) are the effective cross sections for absorption and emission at 1030 nm.

For our calculations, we use measured effective absorption and emission cross sections at 300 K from Koerner et al. [6]. The integration of Eq. (23) and Eq. (24) is done for \(d \gg 1/k_L\) and \(d \gg 1/k_p\) and yields

\[
(\bar{a}d)_{SHB} = \alpha_{p,0} \frac{1}{\alpha_{p,0} + 1} \frac{1}{\alpha_{p,0} + 1} \frac{1}{1 + 2i_{L,0} + 2i_{p,d}Q_{SHB} + 4i_{L,0} + 4i_{p,d}Q_{SHB} + 4i_{L,0}} d \tag{26}
\]

\[
(\bar{a}d)_{PM} = \alpha_{p,0} \frac{2i_{L,0}}{\alpha_{p,0} + 1} \frac{1}{\alpha_{p,0} + 1} \frac{1}{1 + 4i_{L,0} + 4i_{p,d}Q_{PM}} d \tag{27}
\]

In the following, the implications of multipass pumping are investigated. For reasons of simplicity, we assume that the pump light is monochromatic and s-polarized for calculating \((\bar{a}d)_{SHB}\) and \((\bar{a}d)_{PM}\). A disk made from Yb:YAG is typically pumped several times above the saturation intensity of the pump light. Pumping leads to a depletion of the ground-state and the absorption of the pump light is reduced. Above the lasing threshold, the ground state is re-populated by stimulated emission of the laser light. We account for the depletion and the re-population of the ground state by introducing an effective pump intensity. The effective pump intensity is the product of the diode pump intensity \(i_{p,d}\) and the pump enhancement factor \(Q\).
\[ i_{p, \text{noSHB}} = i_{p, d} \frac{1 - \exp\left(-M_P \left(\overline{a d}\right)_{\text{noSHB}}\right)}{(\overline{a d})_{\text{noSHB}}} := Q_{\text{noSHB}} \]

where \( M_P \) is the number of passes of the pump light through the disk. The effective pump intensity for a homogeneously pumped laser in single-frequency operation is given by

\[ i_{p, \text{SHB}} = i_{p, d} \frac{1 - \exp\left(-M_P \left(\overline{a d}\right)_{\text{SHB}}\right)}{(\overline{a d})_{\text{SHB}}} := Q_{\text{SHB}} \]

In the case of phase-matched pumping the effective pump intensity is a function of the axial coordinate \( z \):

\[ i_{p, \text{PM}(z)} = 2i_{p, d} \sin^2(kz) \frac{1 - \exp\left(-M_P \left(\overline{a d}\right)_{\text{PM}}\right)}{(\overline{a d})_{\text{PM}}} := Q_{\text{PM}} \]

These effective intensities are valid for a small single-pass absorption \( \alpha_{p,0}d \ll 1 \). Note that the enhancement factors \( Q_{\text{noSHB}}, Q_{\text{SHB}} \) and \( Q_{\text{PM}} \) are a function of the effective pump intensity as well. Thus, the equations are transcendent and a numerical solution of the pump enhancement factors is required.

In the following the mean gain-length products for the three lasers are calculated. The mean gain-length product \((gd)_{\text{noSHB}}\) of the homogeneously pumped multimode laser is no function of the axial coordinate and given by

\[ (gd)_{\text{noSHB}} = 2\alpha_{L,0} \frac{(B - 1) i_{p, \text{noSHB}} - 1}{1 + i_{p, \text{noSHB}} + 2i_{L,0} d} \]

where \( \alpha_{L,0} \) is the small-signal absorption coefficient at the laser wavelength \( \lambda_L \) which is given by \( \alpha_{L,0} = \sigma_{\text{abs},L} \rho_{\text{dot}} \). The mean gain-length product of the laser that is forced to a single frequency \((gd)_{\text{SHB}}\) and the mean gain-length product of the laser that is pumped by a matched standing wave pattern \((gd)_{\text{PM}}\) is determined by integrating the saturated gain coefficient along the axial coordinate \( z \):
Fig. 3. Output intensity normalized to the saturation intensity $i_{\text{sat}}$ and optical efficiency
$i_{\text{out}}/i_{\text{p,d}}$ of the three lasers. The resonator losses are assumed to be $L_{\text{res}} = 0.2\%$ and the
reflectivity of the back side of the disk is assumed to be $R_d = 1$. The output coupling is
optimized for a diode pump intensity of $i_{\text{p,d}} = 0.3$ for each laser individually. The laser
that is pumped by a phase-matched standing wave pattern has the lowest lasing threshold
and the highest efficiency.

\[
\left(\overline{gd}\right)_{\text{SHB}} = 4\alpha_{L,0} \int_0^d \frac{(B - 1) i_{p,\text{SHB}} - 1}{1 + i_{p,\text{SHB}} + 4i_{L,0} \sin^2(k_{L} z)} \sin^2(k_{L} z) \, dz
\]
\[
\approx \frac{\alpha_{L,0} d}{i_{L,0}} \left[(B - 1) i_{p,\text{SHB}} - 1\right] \left[1 - \left(1 + \frac{4i_{L,0}}{1 + i_{p,\text{SHB}}}\right)^{-1}\right], \text{ for } dk_L \gg 1
\]

\[
\left(\overline{gd}\right)_{\text{PM}} = 4\alpha_{L,0} \int_0^d \frac{(B - 1) i_{p,0} - 1}{1 + (2i_{p,d} Q_{PM} + 4i_{L,0}) \sin^2(k_{PM} z)} \sin^2(k_{PM} z) \, dz
\]
\[
\approx 2\alpha_{L,0} d \left[\frac{2i_{L,0} \left(4i_{L,0} + 2i_{p,d} Q_{PM} + 1\right)^{-1} + i_{p,d} Q_{PM} (B - 1) - 1}{\left(2i_{L,0} + i_{p,d} Q_{PM}\right)^2}\right. \\
\left. + \frac{i_{p,d} Q_{PM} \left(B (4i_{L,0} + 2i_{p,d} Q_{PM} + 1)^{-1} - B + i_{p,d} Q_{PM} (B - 1)\right)}{\left(2i_{L,0} + i_{p,d} Q_{PM}\right)^2}\right], \text{ for } dk_p, dk_L \gg 1
\]

where $k_{PM} = k_{p \cos \theta_{PM}} = k_{L}$ is the angular wavenumber in the case of phase-matched pumping. We have not inserted the effective pump power from Eq. (29) into Eq. (32) for reasons of clarity. In steady state operation above the lasing threshold the round-trip gain equals the loss:

\[
\exp \left(\overline{gd}\right) (1 - T_{OC}) (1 - L_{\text{res}}) R_d = 1
\]
Depending on the type of laser, \( g_d \) in Eq. (34) is replaced by the expressions \((g_d)_{\text{noSHB}}\), \((g_d)_{\text{SHB}}\), or \((g_d)_{\text{PM}}\) from Eq. (31), Eq. (32), or Eq. (33), respectively. \( T_{\text{OC}} \) is the transmission of the output coupler, \( L_{\text{res}} \) denotes the resonator losses due to scattering and impurity absorption, and \( R_d \) is the reflectivity of the HR-coated back side of the disk.

The gain-loss balance Eq. (34) is solved for the circulating laser intensity in the resonator \( i_L \). The explicit analytic solution for \( i_L \) is complex and not presented here. Fig. 3 shows the output powers and optical efficiencies when Eq. (34) is solved by a fixed-point iteration. For each laser, the output coupling was optimized for a diode pump power of \( i_{p,d} = 0.3 \) and \( M_{p} = 16 \) passes through the disk. At this pump power, the laser that is forced to a single mode has the lowest optical efficiency of 53% in comparison to the other two lasers which have an optical efficiency of 61%. Near the lasing threshold, the homogeneously pumped multimode laser has a similar output power than the homogeneously pumped laser which is forced to a single mode. At the highest investigated diode pump power of \( i_{p,d} = 0.3 \) the homogeneously pumped multimode laser has nearly the same optical efficiency as the single frequency laser that is pumped by the matched pump pattern. The laser single frequency laser that is pumped by the phase-matched standing pump wave has an output power that is roughly 15% higher than the output power of the more incoherently pumped laser that is forced to a single mode. The reason for the lower threshold and the better efficiency of the single-frequency laser that is pumped by the phase-matched standing wave is its better saturation of the gain and its lower reabsorption losses. Less gain is lost as fluorescence because the peaks of the standing wave of the laser mode coincide with the spatial peaks of the gain distribution and there is no reabsorption of laser light at the nodes of the standing pump wave.

In practice, the optical efficiency may be smaller due to energy migration in the crystal that might occur especially in highly doped disks. However, experimental investigations of the spatial hole burning carried out with disks having a doping concentration around 10% did not suggest a strong influence of energy migration [7].

4. **Mode competition in the case of the phase-matched pumping**

The mode competition in an Yb:YAG thin-disk laser is simulated based on our previous publication [7]. This simulation iteratively solves the gain-loss balance equation for a given number of modes considering their standing wave patterns. A competition of 300 modes was simulated. These modes were equally spaced in wavelength in the interval from 1027 nm to 1033 nm. A higher number of modes did not change the results.

61 simulations were carried out for different angles of incidence of the pump light. All other quantities were kept constant. The angles of incidence of the pump light ranged from 36° to 40°. The angle of incidence of the pump is coupled to the laser wavelength by Eq. (4). That means the laser oscillates on a mode whose standing wave pattern has the optimal overlap with the standing wave pattern of the pump light. This can be seen in the results of the simulation which are shown in Fig. 4. The black line in the figure indicates the laser wavelength for perfect phase matching. The phase matching condition that is shown by the black line results in laser wavelengths that are significantly shorter than the peak of the spectral gain curve of Yb:YAG if the angle of incidence is larger than 38°. Locking of the laser wavelength by the standing wave of the pump radiation is therefore lost in these regions. The laser trades the advantage of better saturation in the locked state for a better spectral match with the gain. This results in the oscillation of multiple longitudinal modes due to spatial hole burning. The laser oscillates in single-frequency mode in the region from 38.0° to 38.5°. The laser wavelength is locked by the period of the standing wave of the pump and can be tuned by simply changing the angle of incidence of the pump \( \theta_0 \).
Fig. 4. Simulations of an Yb:YAG thin-disk laser that is pumped by partially coherent pump diodes at 969 nm. The polarization of the pump light is random, the emission bandwidth is equal to $\delta \lambda = 0.6$ nm (FWHM), and the width of the angular distribution is equal to the critical width $\delta \theta = \delta \theta_{\text{crit}}$. The pump power is 300 W, the pump spot radius is 2.75 mm, the number of double passes is 16, the output coupling is 2 %, and the round-trip intra-cavity loss is 0.4 %. The red crosses indicate the oscillation of a mode at the respective wavelength. The black line depicts the angle, at which the standing wave pattern of a laser mode is equal to the standing wave pattern of the pump mode. (a) shows the resulting optical spectrum of the laser light and (b) shows the output power as a function of the angle of incidence $\theta_0$. The green regions depicts angles of incidence where the laser operates on a single mode only.

5. Conclusion

We have presented a new approach for tunable single-frequency emission of thin-disk lasers without using etalons or spectral filters. Sufficiently coherent pump diodes lead to a standing wave pattern inside the disk to which the laser wavelength can be locked. The grating period of this standing wave pattern can be tuned by changing the angle of incidence of the pump light on the disk. This not only leads to a tunable single-frequency laser but also to an increase of the power efficiency of about 10 %. We did not have suitable laser diodes to demonstrate this experimentally. However, we did show tuning of the laser wavelength by phase-matched pumping for an intra-cavity pumped thin-disk laser in a previous publication [2]. Now we have shown that this should also be possible for a diode-pumped thin-disk laser. These results can be used to spectrally combine the beams of several thin-disk lasers that are tuned to slightly different wavelengths.

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