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Diffraction analysis of aberrated laser resonators

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ABSTRACT A numerical analysis of laser resonators with aberrations is presented. The analysis shows that aberrations lead to large diffraction losses of laser resonators which are laid out to produce diffraction-limited beam quality. Static or dynamic compensation of the aberrations is possible and would yield much higher output power.

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1 Introduction

The main obstacles in developing high-power solid-state lasers with diffraction-limited beam quality are thermo-optical aberrations of the active medium.

We performed a diffraction analysis of aberrated stable laser resonators based on the Fox and Li algorithm, taking into account the gain saturation of the beam within the active medium. The goal is to understand the influence of aberrations and to assess the potential of adaptive laser resonators in which deformable intra-cavity mirrors are used to compensate for the aberrations.

Several studies were performed to analyze the influence of aberrations on solid-state lasers. Hodgson and Weber [1] applied the Fox and Li algorithm [2] to analyze the influence of spherical aberration on stable resonators. The numerical results showed that the diffraction losses increase for stable resonators operated near the limit of stability. Operating a stable resonator near the limit of stability is necessary in order to obtain a large fundamental mode and diffraction-limited beam quality. Bourderionnet et al. [3] also used the Fox and Li algorithm in order to investigate the influence of spherical aberration and astigmatism on stable resonators. They showed that for typical ratios of the laser beam diameter to the laser rod diameter of 0.5, just 0.5 λ of spherical aberration will create diffraction losses of more than 25%. This is in agreement with our result for -0.5λ of spherical aberration which we will present in Fig. 4. They also found no significant difference between resonator calculations that take gain and gain saturation into account and calculations of empty resonators. Comparing this finding with our calculations of resonators with gain is difficult because we do not know the gain, the saturation intensity, and the output coupling they used. Kennedy [4, 5] considered the problem of a stable resonator with spherical aberration from a very interesting point of view that provided deep physical insight. Resonator mirrors with spherical aberrations can sustain Gauss–Laguerre modes of high azimuthal order. Such modes have an annular footprint on the resonator mirrors. Each mode thus interacts only with an annular region of the mirror and the mirror can be approximated by a spherical surface across this region. Simulating gain competition between such helicoidal modes produced a mode spectrum which depends on the strength of the spherical aberration. He calculated the beam quality by incoherent superposition of the lowest-loss modes. One drawback of this approach is that pure spherical aberration is the only type of aberration that can be analyzed. Another drawback is that even for pure spherical aberration, the eigenmodes of the empty resonator have to be selected a priori and the true eigenmodes and their diffraction losses can not be obtained.

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2 Diffraction analysis

In the case of isotropic laser materials such as Nd:YAG, thermally induced birefringence is the major source of aberrations in cylindrical laser rods. However, it was shown that thermal birefringence can be eliminated in a setup comprising two identical laser rods, a telescope, and a 90°-polarization rotator [6]. This scheme is now frequently employed, and we therefore assume in our simulations that the active medium is free of birefringence. The remaining aberrations of the thermal lens are caused by unavoidable inhomogeneities of the heat source density in the laser crystal, by inhomogeneous cooling of the crystal due to stimulated emission, by the temperature-dependence of the thermal conductivity, and numerous other effects that lead to a non-parabolic temperature profile in the laser rod.

When analyzing wavefront aberrations of a circular pupil, Zernike polynomials [7, 8] are convenient to use because they have some useful properties: they form a complete set, they can be separated into radial and angular functions, and the individual polynomials are orthogonal and normalized over the unit circle. The expansion of a wavefront $W(\varrho, \theta)$ cross a circular aperture in terms of Zernike polynomials can be written

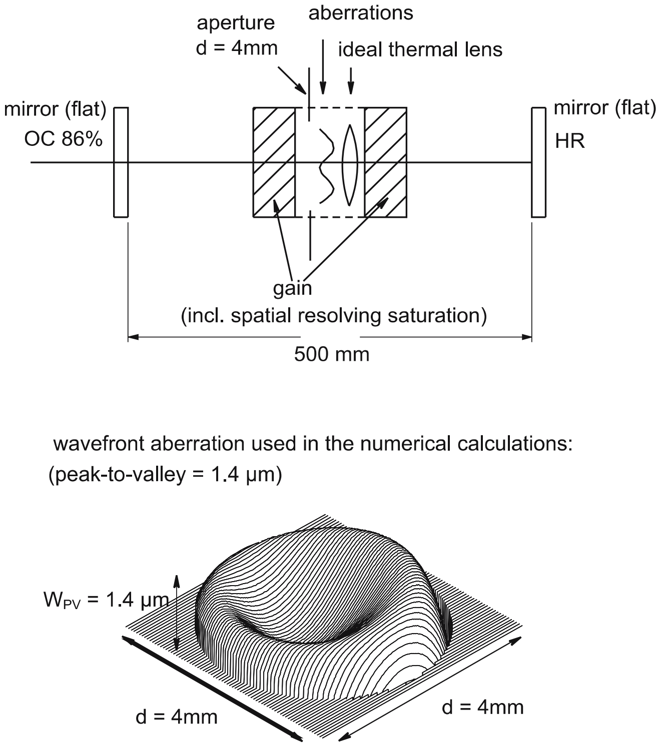

FIGURE 1 *Top*: The resonator model that was used to calculate the eigenmodes of resonators with aberrations by means of the Fox and Li algorithm. The reflectivity of the output mirror was 86%. The limiting aperture is given by the laser rod. *Bottom*: Aberration that was used in the numerical model, measured in an arc-lamp pumped Nd:YAG laser [11]

as follows:

$$W(\varrho,\theta)=\lambda\left[c_{00}+\sum_{n=1}^{\infty}\sum_{l=-n}^{n}c_{nl}Z_n^l\right], \tag{1}$$

$n\geq|l|\,,\ n-|l|=\text{even}$

$$Z_n^l(\varrho,\theta)=R_n^l(\varrho)\begin{Bmatrix}\cos(|l|\,\theta)\text{ when } & l>0\\ \sin(|l|\,\theta)\text{ when } & l<0\end{Bmatrix}. \tag{2}$$

The factors c_{00} and c_{nl} are the Zernike coefficients and Z_n^l are the Zernike polynomials. The variable n is the radial order and the variable l is the azimuthal order of the polynomial. The parameter $\varrho=r/r_0$ is the normalized radial coordinate inside the circular pupil of physical radius r_0. The indexed function $R_n^l(\varrho)$ is the radial polynomial defined by

$$R_n^l(\varrho)=\sum_{s=0}^{\left(\frac{n-|l|}{2}\right)}\frac{(-1)^s(n-s)!}{s!\left(\frac{n+|l|}{2}-s\right)!\left(\frac{n-|l|}{2}-s\right)!}\varrho^{n-2s}. \tag{3}$$

The Zernike polynomials represent a rapidly converging basis set for the low spatial frequency thermo-optic aberrations and surface figure errors typically encountered in laser crystals. If only the six lowest-order Zernike polynomials (piston, tip, tilt, defocus, 0°-astigmatism, 45°-astigmatism) are present in a resonator without gain, the eigenmodes of this resonator are the familiar Gauss–Hermite or Gauss–Laguerre modes. Only the size and thus the diffraction losses of these modes are affected by the six lowest Zernike polynomials. We, therefore, do not consider these polynomials as representing aberrations. However, higher-order Zernike polynomials will result in aspherical resonators which have to be described by entirely different sets of eigenmodes.

Zernike polynomials were developed to describe uniformly illuminated circular pupils of finite radius and this radius is used as the normalization radius r_0. In our simulations the obvious choice for the normalization radius is the radius of the aperture of the laser rod, even though it is not homogeneously illuminated if a Gaussian beam is oscillating. Defining the proper normalization radius is a more serious problem if Zernike aberrations are applied to a beam propagating outside of a laser resonator [9].

Figure 1 shows a schematic diagram of the symmetrical resonator model and of the wavefront aberrations that were used in our numerical calculations. The Fox and Li algorithm for solving the diffraction integral was implemented using the GLAD code (General Laser Analysis and Design) from Applied Optics Research [10]. Gain saturation is taken into account by applying Beer's law with a spatially non-uniform saturation of the laser mode:

$$I_{i,t}(x,y)=I_{(i-1),t}(x,y)\times\exp\left[\frac{g_0}{1+\frac{I_{i,(t-2)}(x,y)+I_{(n_\text{step}-i),(t-1)}(x,y)}{2I_S}}\Delta z\right], \tag{4}$$

$\Delta z=l/n_\text{step}$,
$i=1\ldots n_\text{step}$,
$t=1\ldots(2\times\text{maximum round trips})$,

where g_0 is the unsaturated small-signal gain coefficient and $I(x,y)$ is the optical power per unit area. The area is divided into 1024×1024 elements in order to have sufficient spatial resolution and limit the computational load. The gain medium we wanted to simulate is a high-power Nd:YAG laser rod of $l=180$ mm length which cannot be approximated by a single gain sheet if nonlinear gain saturation is to be taken into account. Therefore, the gain medium was split into $n_\text{step}=100$ sheets and the propagation distance Δz between the gain sheets is thus $\Delta z=l/n_\text{step}$. The small-signal gain coefficient in our model had a value of $g_0=0.05\,\text{cm}^{-1}$, resulting in a small signal gain of $g_0 l=0.90$ for a single pass through the 180 mm long laser rod. I_S is the saturation fluence (we used $I_S=1\,\text{kW/cm}^2$) and $I_{i,t}(x,y)$ is the intensity in the i-th slice and the t-th trip through the gain medium. The output coupling mirror has a reflectivity of Refl = 86% and the wavelength is 1.064 µm. Wavefront aberrations were included in the center of the gain medium. The aberrations which we used in our simulations have actually been measured at a birefringence-compensated industrial arc-lamp pumped Nd:YAG laser using a Shack–Hartmann wavefront sensor [11]. For the numerical analysis, the Zernike polynomials piston, tip, tilt, and defocus were subtracted from the measured wavefront in order to investigate the sole influence of higher-order aberrations. The Zernike polynomials and corresponding coefficients of the aberrations are listed in Table 1. The total peak-to-valley wavefront distortion was 1.4 µm.

Zernike polynomial[a]	common name	Zernike coefficient[b]
$Z_2^{-2} = \rho^2 \sin(2\theta)$	*astigmatism* 0°	0.200
$Z_2^{2} = \rho^2 \cos(2\theta)$	*astigmatism* 45°	−0.500
$Z_3^{-1} = (3\varrho^2 - 2\varrho)\sin(\theta)$	*coma x*	0.050
$Z_3^{1} = (3\varrho^2 - 2\varrho)\cos(\theta)$	*coma y*	−0.135
$Z_4^{0} = 6\varrho^4 - 6\varrho^2 + 1$	*spherical aberration* ϱ^4	−0.130
$Z_6^{0} = 20\varrho^6 - 30\varrho^4 + 12\varrho^2 - 1$	*spherical aberration* ϱ^6	−0.020

[a] The definition are those given by Born and Wolf [7] and the ordering is given by Wyant [8]
[b] Zernike polynomials normalized over a diameter of 4.4 mm. Aberration of a lamp-pumped Nd:YAG laser rod, pumped with 4.9 kW electrical power and, measured with a Shack–Hartmann sensor [11]

TABLE 1 The Zernike polynomials used in the numerical calculation

A resonator with g-parameters g_1 and g_2 that contains a medium with a thermal lens can be treated as a resonator without a thermal lens but different, so-called "equivalent g-parameters" g_1^* and g_2^*. The equivalent g-parameters are [12]:

$$g_j^* = g_j - Dd_k\left(1 - \frac{d_j}{R_j}\right) \qquad j,k = 1,2 \quad j \neq k, \qquad g_j = 1 - \frac{(d_1+d_2)}{R_j}, \tag{5}$$

where d_j is the distance between mirror j and the closest principal plane of the thermal lens, R_j is the radius of curvature of mirror j, and D the refractive power of the thermal lens. We analyzed symmetrical stable resonators with g-parameters $g^* = g_1^* = g_2^*$. Unstable resonators with $|g_1^* g_2^*| > 1$ were not investigated.

The beam quality factor M^2, the diffraction losses per round trip, and the output power were calculated for a large number of stable resonators with g-parameters ranging from $g^* = -1$ to $g^* = 1$. In a real laser with flat resonator mirrors, these g-parameters could be obtained by varying the pump power and thus changing the power of the thermal lens in the active medium or by varying the resonator length. However, in our simulations the pump power and thus the gain were fixed while the power of the thermal lens was varied in order to study the sole influence of aberrations for different g-parameters at constant small-signal gain.

After about 2500 round trips, the mode inside the resonator usually had reached a steady-state. However, even after 2500 round trips, the eigenvalue Λ of some resonators was oscillating between two values which differed by up to 10%. Since these oscillations appeared to be undamped, calculating more round trips would not have eliminated the oscillations. This is a well-known problem of the Fox and Li algorithm. The eigenvalue Λ represents the fraction of the total power that is not lost in each round trip due to output coupling and clipping at the laser rod aperture of 4 mm diameter. $L = \Lambda/\text{Refl}$ is thus the power transmission per round trip of the laser rod aperture. The value $\Delta L = 1 - L$ is usually called the diffraction loss or clipping loss of the resonator per round trip.

The output power of a laser strongly depends on the diffraction losses because the gain saturation depends on the intra-cavity power. The errors in our calculated values of the laser output power can therefore be larger than 10%. Great care was taken to avoid aliasing due to the Fast-Fourier-Transform algorithm of the GLAD code and to avoid insufficient spatial resolution of the numerical fields. The first round trips of each resonator calculation were monitored in order to ensure that these effects were not present and a different field size or a different propagation algorithm was selected if necessary. Some physical approximations are inherent to the Fox and Li algorithm. For example, spatial hole burning, relaxation oscillations, and superposition of incoherent transverse modes are not represented. Nevertheless, the algorithm is known to produce meaningful results for the diffraction losses, the output power, and the beam quality of real laser resonators.

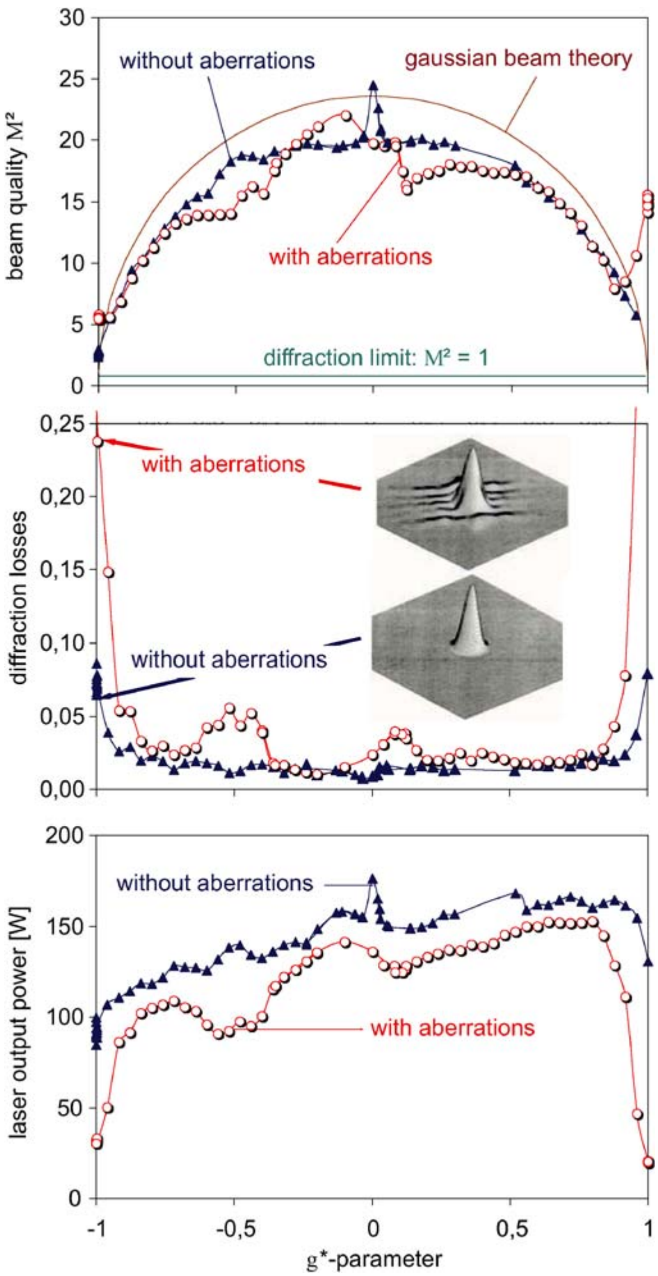

FIGURE 2 Beam quality factor M^2, diffraction losses, and output power with and without aberrations for g-parameters ranging from $g^* = -1$ to 1

The calculated beam quality factors M^2 are prone to numerical errors. M^2 is based on the second moments of the near- and far-field intensity distributions, and it is well-known that the second moments are prone to errors because the intensities are weighed by the square of the distance of the center of gravity. The most important results of our calculations are the diffraction losses of the different resonators.

In Fig. 2, the M^2 values, the diffraction losses ΔL, and the laser output power of all resonators are shown as a function of the g-parameter. If aberrations and gain saturation were neglected, the Laguerre–Gaussian modes were the eigenmodes and the beam quality could be estimated by the ratio between the square of the laser rod radius and the radius of the fundamental Laguerre–Gaussian mode in the laser rod. The resulting beam quality factors M^2 are shown for comparison.

By looking at the top of Fig. 2, it seems that the aberrations apparently have little influence on beam quality. Only in the case of a near-planar resonator ($g^* \approx 1$) and an almost concentric resonator ($g^* \approx -1$) beam quality is deteriorating due to the aberrations. The intensity distribution on the output coupling mirror shows side lobes if aberrations occur.

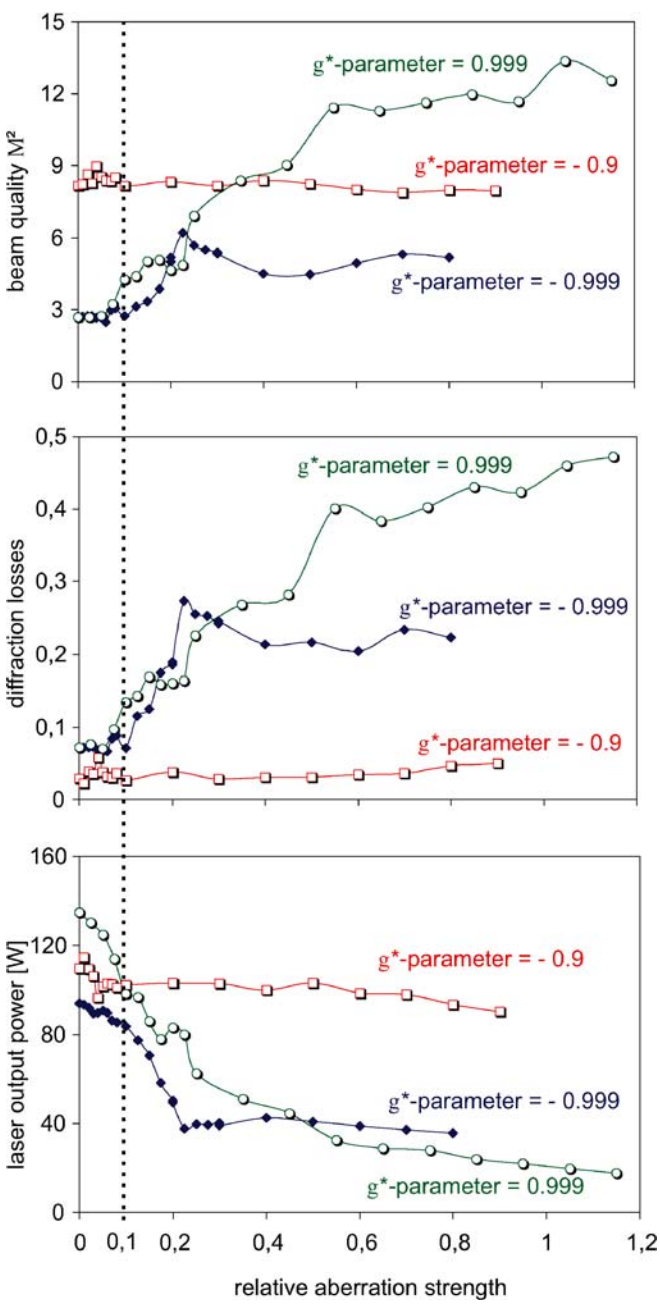

FIGURE 3 Beam quality factor M^2, diffraction losses, and output power of three resonators with different g-parameters g^* as a function of the relative aberration strength

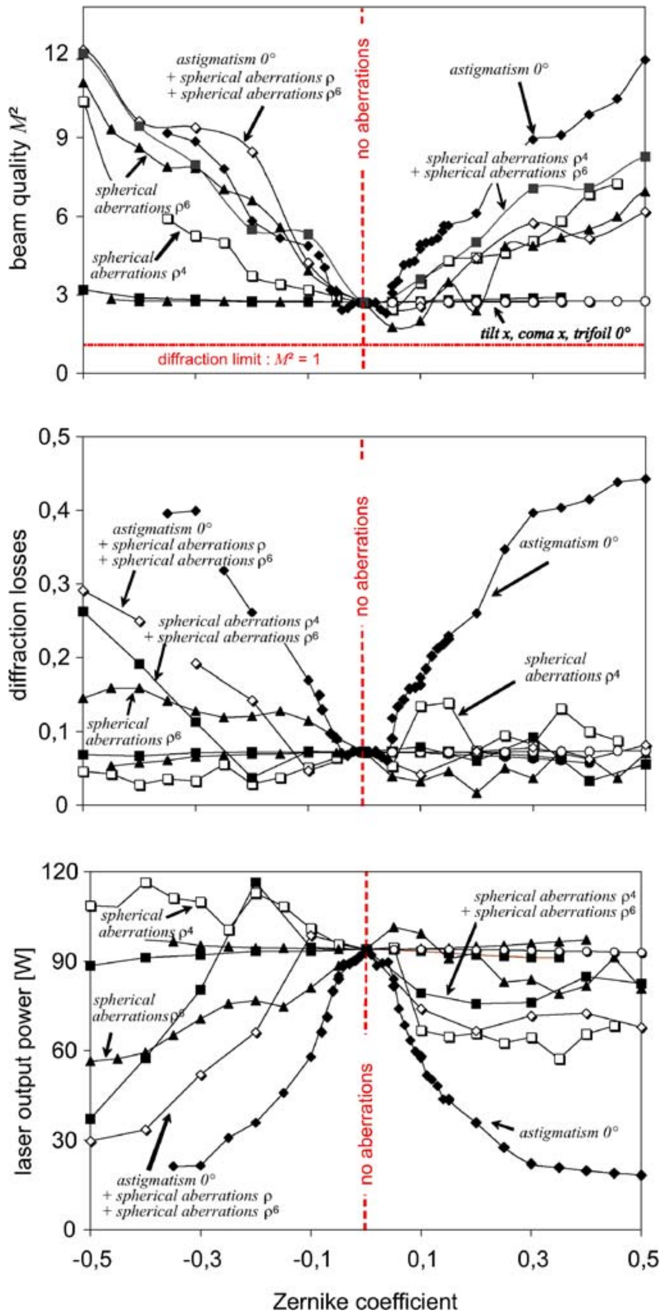

FIGURE 4 Beam quality factor M^2, diffraction losses, and output power of a resonator with g-parameter of $g^* = -0.999$ as a function of Zernike coefficient for different Zernike polynomials. The resonator without aberrations is located at a Zernike coefficient of zero and marked with the *dashed line*

The most important result of our calculations is that in the regions of fairly good beam quality near the resonator stability limits at $g^* = \pm 1$, the diffraction losses are very high and the output power drops to low values. We can conclude that aberrations only have a significant influence if the resonator is operated near the geometrical stability limits. Here, aberrations result in strongly increased diffraction losses and prevent efficient laser operation. On the other hand, good beam quality can only be achieved near the stability limits, where the fundamental mode diameter is large. Thus it is not possible to obtain high output power and good beam quality simultaneously in the presence of severe aberrations.

Since we want to correct the aberrations with a deformable mirror, it is important to know the necessary precision of the surface figure of a deformable mirror. We, therefore, performed another set of calculations in which the aberration strengths were varied for three resonators with g-parameters of $g^* = -0.999$, $g^* = 0.999$, and $g^* = -0.9$. The measured aberration shown in the lower part of Fig. 1 was simply multiplied by a factor (termed "relative aberration strength") ranging from 0 to 1.2. The results of these calculations are shown in Fig. 3. Diffraction losses start to increase when the relative aberration strength becomes larger than about 0.1 for nearly diffraction limited laser beams with $g^* = -0.999$ and $g^* = 0.999$. In contrast, at $g^* = -0.9$ the resonator already operates in transverse multimode and the aberrations have little influence. Remembering that the peak-to-valley variation of our original aberration is 1.4 μm (see Fig. 1), this result means that we need to compensate the wavefront distortions to an accuracy of roughly 0.14 μm.

The last part of the simulations comprised a resonator with g-parameter of $g^* = -0.999$ onto which different Zernike aberrations are acting. The results for pure and mixed Zernike polynomial aberrations are shown in Fig. 4. It can be seen that aberrations like tilt, coma and trifoil have almost no effect on beam quality and diffraction losses for the investigated resonator with the g-parameter $g^* = -0.999$. In contrast, other Zernike polynomial aberrations such as astigmatism and spherical aberration ϱ^4 have a strong influence. They lead to increased diffraction losses and degradation of beam quality.

In some parts of the diagram, aberrations even have the ability to decrease diffraction losses. This effect is observed for spherical aberration ϱ^6 in the range of Zernike coefficients between 0 to 0.5 and for spherical aberration ϱ^4 in the range of Zernike coefficients between −0.5 to 0.

The simulations were done for pure Zernike polynomials as well as for superpositions of different Zernike polynomials. Figure 4 shows that the combined influence of different Zernike polynomials is not just the sum of the influences of the individual Zernike polynomials. This is to be expected because a laser resonator is a nonlinear system. For example, a combination of Zernike coefficients of 0.1 for astigmatism, spherical aberrations ϱ^4 and spherical aberrations ϱ^6, leads to a weak reduction of the diffraction losses and an improvement of beam quality, while 0.1 waves of astigmatism or spherical aberrations ϱ^4 lead to a strong increase of the diffraction losses.

3 Summary

In summary, the numerical calculations show that aberrations have a significant influence if the resonator is operated near the stability limits. In these cases, the aberrations result in strongly increased diffraction losses and prevent efficient laser operation. It is important to note that a resonator has to be operated near the stability limits if good beam quality is required. Some aberrations like tilt, coma or trifoil have almost no effect on diffraction losses and beam quality, while astigmatism and spherical aberration ϱ^4 have a strong influence and lead to increased diffraction losses. Adaptive optics has the ability to compensate the aberrations. A deformable mirror with an accuracy of roughly 0.1 μm, equivalent to $\sim \lambda/10$, is necessary.

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